**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

Solution –

from scipy.stats import norm

nd = norm(45, 8)

z1 = 1 - nd.cdf(50)

z1

1. 0.3875
2. **0.2676 - Answer**
3. 0.5
4. 0.6987
5. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
6. More employees at the processing center are older than 44 than between 38 and 44.

**False – The approx. number of employees between age 38 and 44 are – 136**

**The approx. number of employees above age 44 are – 63**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**True**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

2X1 = N(2μ, 4σ2) and X1 + X2 = N(2μ, 2σ2)

Difference between two iids N1(μ1, σ12) and N2(μ2, σ22) is N(μ1 - μ2, σ12 + σ22).

So 2X­1 – (X­1 + X2) = N(2μ - 2μ, 4σ2 + 2σ2) = **N(0, 6σ2)**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

Solution – Should go more than two standard deviations almost 3 standard deviations to get a probability of 0.99. So the range should be close to (40, 160)

1. 90.5, 105.9
2. 80.2, 119.8
3. 22, 78
4. **48.5, 151.5 - Answer**
5. 90.1, 109.9
6. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
7. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Total profit distribution = N(12, 5).

For 95% probability two standard deviations around the mean are needed. So, the range is (2, 22). **In rupees – (2 \* 106 \* 45, 22 \* 106 \* 45) = (9 crores, 99 crores)**

1. Specify the 5th percentile of profit (in Rupees) for the company

16.99 crores of rupees

1. Which of the two divisions has a larger probability of making a loss in a given year?

The 1st division has a probability of 0.04779 of making a loss and the 2nd division has a probability of 0.04005 of making a loss in a given year. So the **1st division** makes more loss.